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THE EFFECT OF MATERIAL HETEROGENEITY  
IN CURVED COMPOSITE BEAMS  
FOR USE IN AIRCRAFT STRUCTURES<sup>1</sup>

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ABSTRACT

A design tool is presented for predicting the effect of material heterogeneity on the performance of curved composite beams for use in aircraft fuselage structures. Material heterogeneity can be induced during processes such as sheet forming and stretch forming of thermoplastic composites. This heterogeneity can be introduced in the form of fiber realignment and spreading during the manufacturing process causing a gradient in material properties in both the radial and tangential directions. The analysis procedure uses a separate two-dimensional elasticity solution for the stresses in the flanges and web sections of the beam. The separate solutions are coupled by requiring that forces and displacements match at the section boundaries. Analysis is performed for curved beams loaded in pure bending and uniform pressure. The beams can be of any general cross-section such as a hat, T-, I-, or J-beam. Preliminary results show that the geometry of the beam dictates the effect of heterogeneity on performance. Heterogeneity plays a much larger role in beams with a small average radius to depth ratio,  $R/t$ , where  $R$  is the average radius of the beam and  $t$  is the difference between the inside and outside radius. Results of the analysis are in the form of stresses and displacements, and they are compared to both mechanics of materials and numerical solutions obtained using finite element analysis.

INTRODUCTION

The use of composite materials in commercial aircraft has been focused on secondary structures such as control surfaces and trailing edge panels. Breakthroughs in manufacturing techniques, materials, and structural concepts are needed so that more primary structures can be produced from composites resulting in structural weight savings, part count reduction, and cost reduction. This research investigates the possibilities of combining a new material system of long discontinuous fibers in a thermoplastic matrix with fabrication techniques such as sheet forming and stretch forming to produce curved beams for use as primary structures in commercial aircraft.

Manufacturing processes such as sheet forming and stretch forming can be used to produce several types of composite parts [1, 2]. The use of a long discontinuous fiber material system allows for material stretching over complex curvature parts while maintaining a high percentage of the continuous fiber material properties [3]. Combination of these forming methods and material system allows the production of complex structures such as curved beams as shown in Figure 1. The microstructure of a curved beam is sensitive to the production method and gradients in material properties are expected in both sheet formed [4] and stretch formed [2] beams. Schematic examples of two types of heterogeneity are shown in Figure

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2; analysis of these types of beams can be useful in determining the effect of such property gradients on the overall performance of a given beam.

Two separate analyses were conducted. The first uses a closed form stress potential approach to investigate the effect of radial heterogeneity on curved beams loaded in pure bending. The stress state is found for beams which can have several different geometries including I, J, T, and rectangular cross-sections. Material properties can be specified independently for each section of the beam, i.e., flange and web can have different properties. Each section of the beam is treated as an individual curved rectangular beam loaded in pure bending and with a constant distributed load on the curved surfaces. Superposition is used to combine the results of the individual sections into the total beam solution. Details of the analysis are provided and results are shown for comparison with known solutions.

The second analysis technique uses a Rayleigh-Ritz approach to solve the minimum potential energy equation in a circular ring loaded with an internal and external pressure. This is an approximate solution which uses an assumed series formulation of the displacement field. The advantage of this method is that it allows for any type of material heterogeneity and can be used to solve other relevant problems such as tensile loaded beams or beams with geometric stress concentrations such as cutouts.

Analysis results are compared to solutions found by using mechanics of materials and finite element analysis. The mechanics of materials solutions are useful for comparing results for beams with homogeneous material properties and the finite element analysis is necessary to solve the problem when the beam has heterogeneous material properties. The first type of analysis has been incorporated into a design tool for analyzing curved beams loaded in pure bending. A wide range of geometric parameters and material properties can be analyzed with relative ease. The second type of analysis is being developed so that a similar tool can be used to analyze curved beams with different loading conditions or geometric configurations.

## ANALYSIS PROCEDURES

### Radial Heterogeneity Analysis

The state of stress and strain is determined for a curved beam loaded in pure bending which has any of the following cross-sections; I-beam, T-beam, J-beam, etc. The solution is found by separating the beam into three sections; each with an applied bending moment and distributed load. A stress potential approach is used to solve the two-dimensional problem in each section. The constitutive relations take the form

$$\epsilon_i = \alpha_{ij} r^{-n} \sigma_j; \quad i, j = 1, 2 \quad (1)$$

where  $\epsilon$  is the two-dimensional strain vector in polar coordinates,  $\sigma$  is the corresponding stress vector,  $r$  is the radial position, and  $\alpha_{ij}$  are the base values of the elements of the compliance matrix;

$$\alpha_{11} = a^n/E_{11} \quad , \quad \alpha_{12} = \alpha_{21} = -\nu_{12} a^n/E_{12} \quad , \quad \alpha_{22} = a^n/E_{22} \quad , \quad (2)$$

and  $a$  is the inside radius of the beam. The degree of radial heterogeneity,  $n$ , allows for a property gradient in the radial direction of the beam. A positive ' $n$ ' defines a beam which is stiffer with increasing radius, a negative ' $n$ ' defines a beam which is more compliant with increasing radius and homogeneous material properties are specified by letting  $n = 0$ . Base values for the material properties are defined along the inside radius of the beam. This constitutive relation, together with equilibrium and compatibility can be combined to form the equation

$$\nabla^4 \phi = 0 \quad (3)$$

where  $\phi$  is the stress potential. We can solve for  $\phi$  by applying boundary conditions to the two-dimensional curved beam as shown in Figure 3; the tractions along the straight edges are represented by a bending moment,  $M$ , and the curved surfaces are traction free. The resulting stresses are [5]:

$$\begin{aligned}\sigma_r &= -\frac{M}{b^2hg} \left[ \frac{c^t - c^{n+1}}{c^s - c^t} \rho^{s-1} + \frac{c^{n+1} - c^s}{c^s - c^t} \rho^{t-1} + \rho^n \right], \\ \sigma_\theta &= -\frac{M}{b^2hg} \left[ \frac{c^t - c^{n+1}}{c^s - c^t} s \rho^{s-1} + \frac{c^{n+1} - c^s}{c^s - c^t} t \rho^{t-1} + (n+1) \rho^n \right],\end{aligned}\quad (4)$$

where,

$$g = \frac{(c^t - c^{n+1})(1 - c^{s+1})}{c^s - c^t} \frac{s}{s+1} + \frac{(c^{n+1} - c^s)(1 - c^{t+1})}{c^s - c^t} \frac{t}{t+1} + \frac{(n+1)}{(n+2)} (1 - c^{n+2}), \quad (5)$$

$$\begin{pmatrix} s \\ t \end{pmatrix} = \frac{1}{2} \left( n \pm \sqrt{n^2 + 4\gamma_n} \right), \quad \gamma_n = \frac{(\alpha_{11} + n \alpha_{12})}{\alpha_{22}}, \quad (6)$$

and  $h$  is the beam thickness,  $c$  is the ratio of the inside radius to the outside radius ( $c = a/b$ ), and  $\rho$  is the ratio of radial position to outside radius ( $\rho = r/b$ ). Notice that the solution is axisymmetric and  $\sigma_{r\theta} = 0$  everywhere.

Another loading condition that produces an axisymmetric state of stress in a curved beam is the classic Lamé's problem, which is a circular cylinder with an internal and external pressure. The stresses in such a cylinder are [5]:

$$\begin{aligned}\sigma_r &= -P c \frac{\rho^{s-1} - \rho^{t-1}}{c^s - c^t} + Q \frac{c^t \rho^{s-1} - c^s \rho^{t-1}}{c^s - c^t} \\ \sigma_\theta &= -P c \frac{s \rho^{s-1} - t \rho^{t-1}}{c^s - c^t} + Q \frac{s c^t \rho^{s-1} - t c^s \rho^{t-1}}{c^s - c^t}\end{aligned}\quad (7)$$

where  $P$  is the internal pressure,  $Q$  is the external pressure, and all the other variables are the same as in the pure bending case. When looking at a section of the cylinder, as shown in Figure 4, the straight edges are not traction free; the tractions can be represented by an end moment and an end load analogous to hoop stress found in a thin walled cylinder. The end load,  $L$ , is determined by integrating the tangential stress across the depth of the beam and the end moment,  $M_L$ , is found by integrating the tangential stress times the radius across the depth of the beam.

The displacements for both of these loading conditions are found using a two step procedure. The first step finds the radial and tangential strains by substituting equations (4) and (7) into equation (1). Then the expressions for the displacements can be found by applying the strain displacement relations [5].

### Superposition of Two-Dimensional Solutions

Now that the solution for the stresses has been established in each individual section under the general loading shown in Figure 5, superposition is used to find the solution of the entire beam. The

curved I-beam, for example, loaded with a bending moment,  $M$ , is separated into three sections with the following bending moments and distributed loads:  $M_1, M_2, M_3, P_2, P_3, Q_1$ , and  $Q_2$ , as shown in Figure 5. Applying superposition; the sum of the moments on the ends must be equal to  $M$ :

$$M_1 + M_2 + M_3 + M_{L1} + M_{L2} + M_{L3} = M \quad (8)$$

where  $M_1, M_2$ , and  $M_3$  are applied bending moments and  $M_{L1}, M_{L2}$ , and  $M_{L3}$  are the bending moments due to the applied distributed loads  $Q_1, P_2$  and  $Q_2$ , and  $P_3$ , respectively.

Six more equations are necessary to solve this problem. The sections must be in equilibrium where they meet, therefore the radial loads must be equal resulting in the following relations:

$$P_2 h_2 = Q_1 h_1 \quad \text{and} \quad P_3 h_3 = Q_2 h_2 \quad (9)$$

where  $h_1, h_2$ , and  $h_3$  are the thickness of each section and the  $P$ 's and  $Q$ 's are the applied pressures. The final equations are found by requiring the continuity of the displacements at the section boundaries. The radial and tangential displacements of section 1 must be equal to the corresponding displacements of section 2 at the section boundary where  $r = b$ . Similar conditions hold at the other section boundary where  $r = c$ .

$$\begin{aligned} u_r^{(1)} &= u_r^{(2)}, \quad \text{at } r = b & u_r^{(2)} &= u_r^{(3)}, \quad \text{at } r = c \\ u_\theta^{(1)} &= u_\theta^{(2)}, \quad \text{at } r = b & u_\theta^{(2)} &= u_\theta^{(3)}, \quad \text{at } r = c. \end{aligned} \quad (10)$$

Equation (8) which is the superposition equation, equations (9) which are the two equilibrium equations, and equations (10) which are the four continuity equations are solved simultaneously for the seven unknowns;  $M_1, M_2, M_3, P_2, P_3, Q_1$ , and  $Q_2$ . The stresses, strains and displacements can be found in each section based on these loading conditions.

### Rayleigh-Ritz Structural Analysis

This method is used to solve the problem of a circular ring loaded by internal and external pressure. It makes use of an assumed displacement field which can also be used to solve several other problems [6]. This method allows for the calculation of stresses in components without the need for elaborate pre- and post-processing; which is especially convenient for parts with complex heterogeneous material properties and geometry.

The principle of minimum potential energy states that of all displacement fields which satisfy the prescribed constraint conditions, the correct state is that which makes the total potential energy,  $\Pi$ , of the structure a minimum [7]. The potential energy of the structure is the sum of the elastic strain energy,  $U$ , and the potential of the external forces,  $V$ . The minimum potential energy is found by setting its first variation equal to zero,  $\delta \Pi = \delta U + \delta V = 0$ ; which can be expanded to

$$\int_A \delta \{\epsilon\}^T \{N\} dA - \int_S \delta \{u\}^T \{t\} dS = 0. \quad (11)$$

where,

- $\{\epsilon\}$  = strain vector
- $\{N\}$  = stress resultant vector
- $\{u\}$  = displacement vector
- $\{t\}$  = applied surface traction vector

and  $A$  is the area of the circular ring and  $S$  is the curve which defines its boundary. We assume the following form of the displacement field,

$$u_r = \sum_{j=0}^N \sum_{k=1}^M q_j^k r^{2(k-M)+1} \cos(2j\theta) \quad , \quad u_\theta = \sum_{j=0}^N \sum_{k=1}^M q_j^{M+k} r^{2(k-M)+1} \sin(2j\theta) \quad , \quad (12)$$

where  $u_r$  and  $u_\theta$  are the displacement components and  $q_j^k$  are unknown parameters. This displacement field satisfies symmetry conditions and, for the case of an isotropic circular ring loaded by internal and external pressure, it converges to the exact solution with very few terms of the series;  $j=0$  and  $k=2$ . Substituting this equation into the principle of minimum potential energy, equation (11), leads to a system of  $M(2N+1)$  linear equations which are solved simultaneously for the unknown parameters,  $q_j^k$ . A detailed description of the solution procedure is presented by Russell [6].

## RESULTS

The superposition model, which is used to find stresses and displacements in a curved beam loaded in pure bending, has been verified by comparing results with mechanics of materials and finite element analysis solutions. Several example problems of isotropic beams having I-, T-, or rectangular cross-sections have been examined and the difference between the superposition and mechanics of materials solutions is less than 1% for all cases. Two-dimensional finite element analysis is used to compare results for a curved heterogeneous anisotropic J-beam. The heterogeneity is introduced into the finite element analysis by varying the material properties in each element of the model. Table 1 compares the superposition results with those found using finite element analysis for a beam with the following dimensions: inside radius is 37.4 inches, the outside radius is 39.9 inches, the lower flange is 0.49 inches wide, the upper flange is 0.89 inches wide and the web and flanges are 0.06 inches thick. The flanges are incorporated into the finite element model by setting the thickness of the inside and outside row of elements accordingly. Three different constitutive relations are examined; the degree of radial heterogeneity,  $n$ , is set equal to -2, 0, and +2, where an 'n' value of -2 corresponds to a beam which is approximately 20% stiffer on the inside radius, an 'n' value of +2 is roughly equivalent to a beam which is 20% stiffer on the outside radius, and an 'n' value of zero means the beam is homogeneous. The finite element analysis results are within 3.4% of the superposition results as shown in Table 1.

The validity of the model has been demonstrated and the effect of radial heterogeneity on beam performance can now be determined. The maximum tangential stress and maximum displacement versus heterogeneity are found for a curved J-beam loaded in pure bending. These maximum values are plotted for several different beam geometries in Figure 6. The degree of heterogeneity is varied from -2 to +2 corresponding to approximately a 20% decrease or 20% increase in stiffness, respectively. The effect of material heterogeneity is highly dependent on the beam geometry which is characterized by the average radius to depth ratio,  $R/t$ ; where  $R = (r_i + r_o)/2$  and  $t = r_o - r_i$ . Heterogeneity has a considerable effect on the maximum tangential stress in beams with a small curvature,  $R/t = 1$ , while it has virtually no effect on the stresses in beams with a large curvature. The maximum displacement is effected by heterogeneity for all beam geometries considered, but, the effect is again seen more drastically in beams with small curvature.

This analysis procedure can be used as a simple tool for preliminary design of curved beams. Given the basic beam dimensions, i.e., inner and outer radii, a range of values for all other dimensions can be selected. Flange widths and thicknesses can be varied independently as well as the material properties and degree of heterogeneity in each section. The results of a sample preliminary design are presented in Table 2. Two types of beams are analyzed; a J-beam with an  $R/t$  ratio of 14.5 and a channel beam with an  $R/t$  ratio of 6.7. The table shows the change in maximum and minimum tangential stress as well as the

maximum deflection for a range of several variables. These variables are the degree of radial heterogeneity which is varied from -2 to +2 for isotropic and unidirectional beams, the inner flange thickness,  $h_1$ , which is varied from 0.09 to 0.89 inches, and the web thickness,  $h_2$ , which is varied from 0.04 to 0.1 inches.

Table 1: Comparison of Superposition and Finite Element Analysis Results for a Heterogeneous, Anisotropic J-Beam Loaded in Pure Bending

Solution Procedure	Degree of Heterogeneity (n)	Maximum Displacement (in)	Maximum Stress (psi)	Minimum Stress (psi)
FEA	-2	7.08 E-5	5.14	-3.50
Superposition	-2	7.21 E-5	5.17	-3.54
% Difference	---	1.8 %	0.6 %	1.1 %
FEA	0	6.53 E-5	5.01	-3.57
Superposition	0	6.76 E-5	5.04	-3.63
% Difference	---	3.4 %	0.6 %	1.7 %
FEA	2	6.23E-5	4.89	-3.67
Superposition	2	6.34 E-5	4.91	-3.71
% Difference	---	1.7 %	0.4 %	1.1 %

Table 2: Design Study Results

Beam Type	Material	Variable Parameter	% Change in Max. Stress	% Change in Min. Stress	% Change in Max. Deflection
J-Beam	Uni-Directional	n = -2 to +2	6.7	6.1	12.8
J-Beam	Isotropic	n = -2 to +2	6.7	6.1	12.8
J-Beam	Uni-Directional	$h_1 = .09$ to $.89$	49.7	20.1	37.4
J-Beam	Uni-Directional	$h_2 = .04$ to $0.1$	33.9	26.7	31.1
C-Beam	Uni-Directional	n = -2 to +2	16.5	14.3	28.4
C-Beam	Isotropic	n = -2 to +2	16.3	14.0	28.2
C-Beam	Uni-Directional	$h_1 = .15$ to $.45$	26.2	10.1	18.7
C-Beam	Isotropic	$h_1 = .15$ to $.45$	26.1	10.2	19.6

The Rayleigh-Ritz technique is used to solve the problem of a curved beam loaded by internal and external pressure. Solutions are compared with exact results for isotropic and axisymmetric anisotropic beams [5], and the difference is within 0.1%. This solution technique is also verified by solving the problem of an infinite plate with a centrally located hole loaded only by an internal pressure where the principal material directions are along the cartesian axes. This problem is modeled by letting  $r_i = 1$  inch,  $r_o = 30$  inches,  $P_i = 1$  psi, and  $P_o = 0$  psi. The stress concentrations found at  $\theta = 0^\circ$  and  $90^\circ$  are within 1% of those found by Lekhnitskii [5]. A carbon reinforced thermoplastic composite ring with an inner radius of 6

inches and an outer radius of 8 inches is analyzed for two different fiber arrangements; one with tangentially oriented fibers and the second with fibers aligned in the x-direction. The stress distribution is axisymmetric in the ring with tangentially oriented fibers as shown in Figure 7a while the ring with straight fibers in the x-direction has a slight stress concentration at approximately  $\theta = 45^\circ$  as shown in Figure 7b. These results are evidence that the tangential heterogeneity due to non-axisymmetric fiber distribution can affect the stresses in a curved beam loaded by internal and external pressure.

## DISCUSSION

A closed form elasticity solution can be used to solve for the stresses and displacements in a heterogeneous anisotropic curved beam loaded in pure bending. The elasticity analysis, based on the superposition of several two-dimensional solutions, provides results which are in very good agreement with those found from mechanics of materials and finite element analysis. The heterogeneity is introduced into the model by defining the material properties as an exponential function of the radius, while the actual heterogeneity due to fiber realignment during forming can be determined using enhanced ultrasonic C-scanning techniques.

The effect of radial heterogeneity on curved beams loaded in pure bending depends on the geometry of the beam. The maximum stress and deflection in beams with a small average radius to depth ratio is significantly affected by heterogeneous material properties. A beam whose stiffness decreases by 20% from the inside to outside radius (i.e.,  $n = -2$ ), shows a 28% increase in the maximum tangential stress and a 75% increase in the maximum deflection when compared to a homogeneous beam if  $R/t = 2$ , but only a 1% and 4% increase, respectively, if  $R/t = 10$ . It is unlikely that radial heterogeneity affects the performance of most beams used in transport aircraft fuselage applications since they have an  $R/t > 10$ ; but this heterogeneity could play a part in the performance of beams used in other applications.

The superposition elasticity analysis has been incorporated into a computer program which can be used for design studies of curved beams. Several of the beam parameters can be varied to determine their overall effect on maximum tensile and compressive stresses, as well as maximum deflections. The variable parameters are the thickness and depth of the flange and web along with their material properties and degree of radial heterogeneity. This provides a quick and easy way to perform initial beam sizing calculations.

The Rayleigh-Ritz analysis can be used to solve problems with both radial and tangential heterogeneity. The importance of this ability is demonstrated by the results of the pressurized ring problem. Isotropic and axisymmetric anisotropic rings have an axisymmetric state of stress when pressurized. Rings with tangential heterogeneity, however, do not have an axisymmetric state of stress when pressurized. Stress concentrations develop which are a function of both the material properties and the heterogeneity. This type of analysis is currently being used to study the effect of heterogeneity on curved beams subject to several different loading conditions; pure bending, internal and external pressure, and end loading. Geometric heterogeneity, such as a notch or cut-out, is also under investigation. Future work includes applying an appropriate failure criterion to the results of these analyses and comparisons with experimental data.

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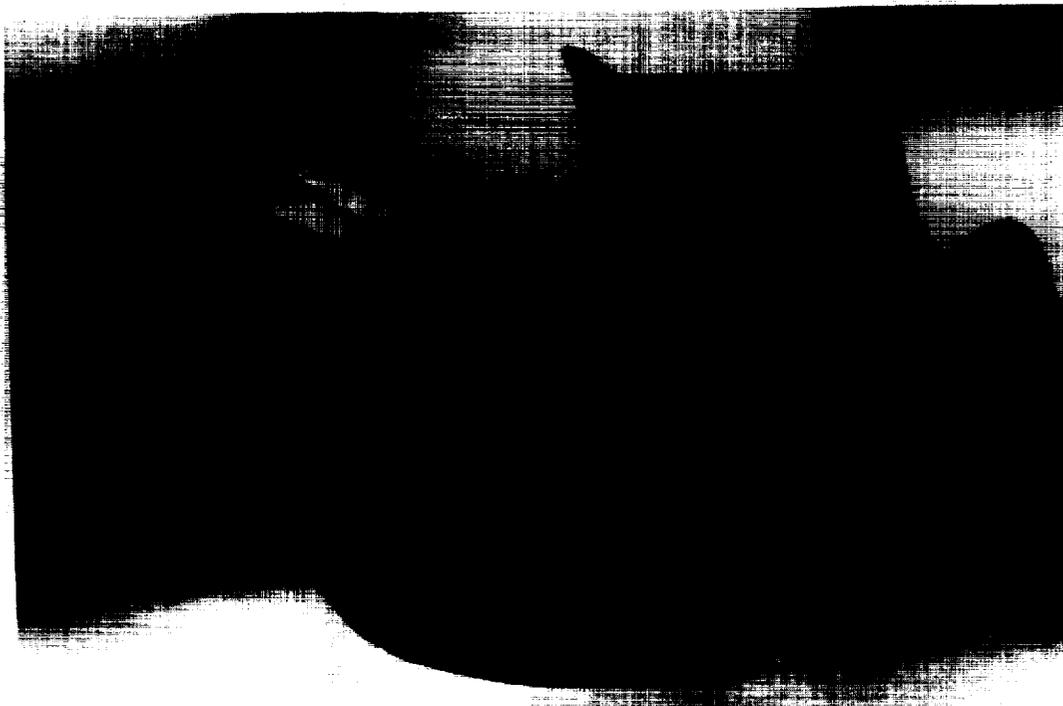


Figure 1: Thermoplastic Composite Curved Beam

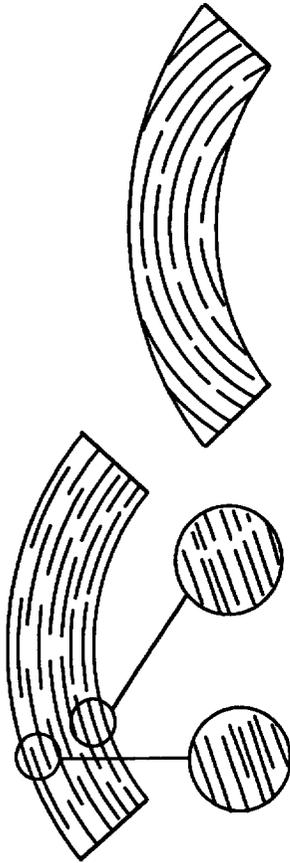


Figure 2: Different Types of Material Heterogeneity

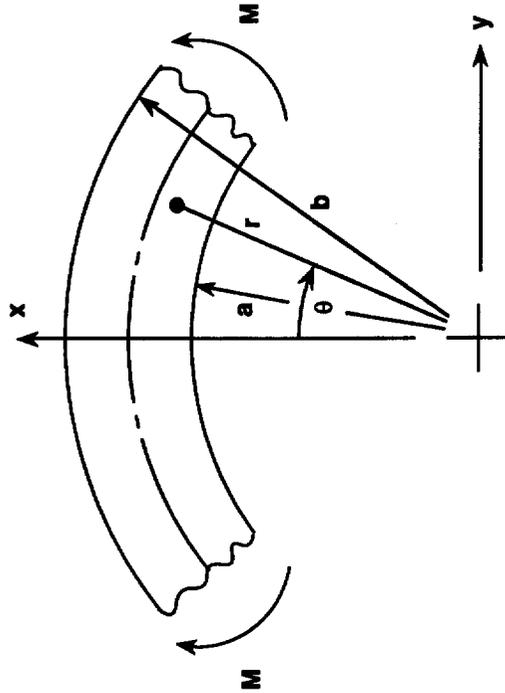
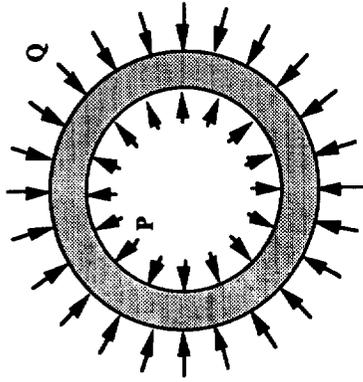


Figure 4: Lame's Problem

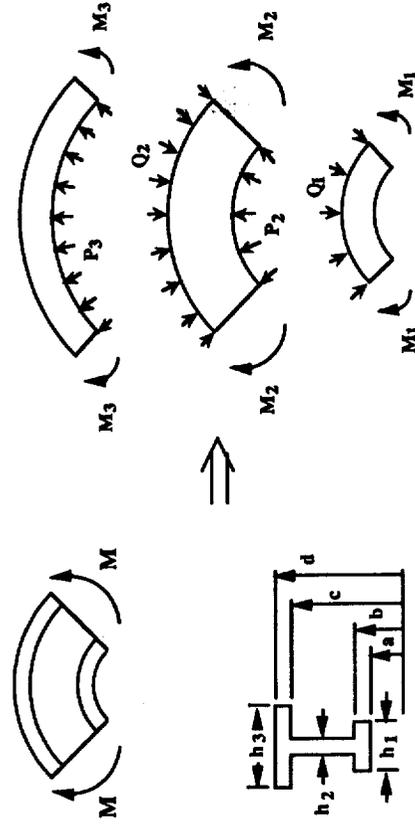
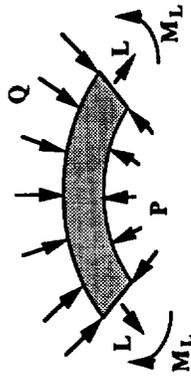


Figure 3: Pure Bending Load Case

Figure 5: Superposition Analysis

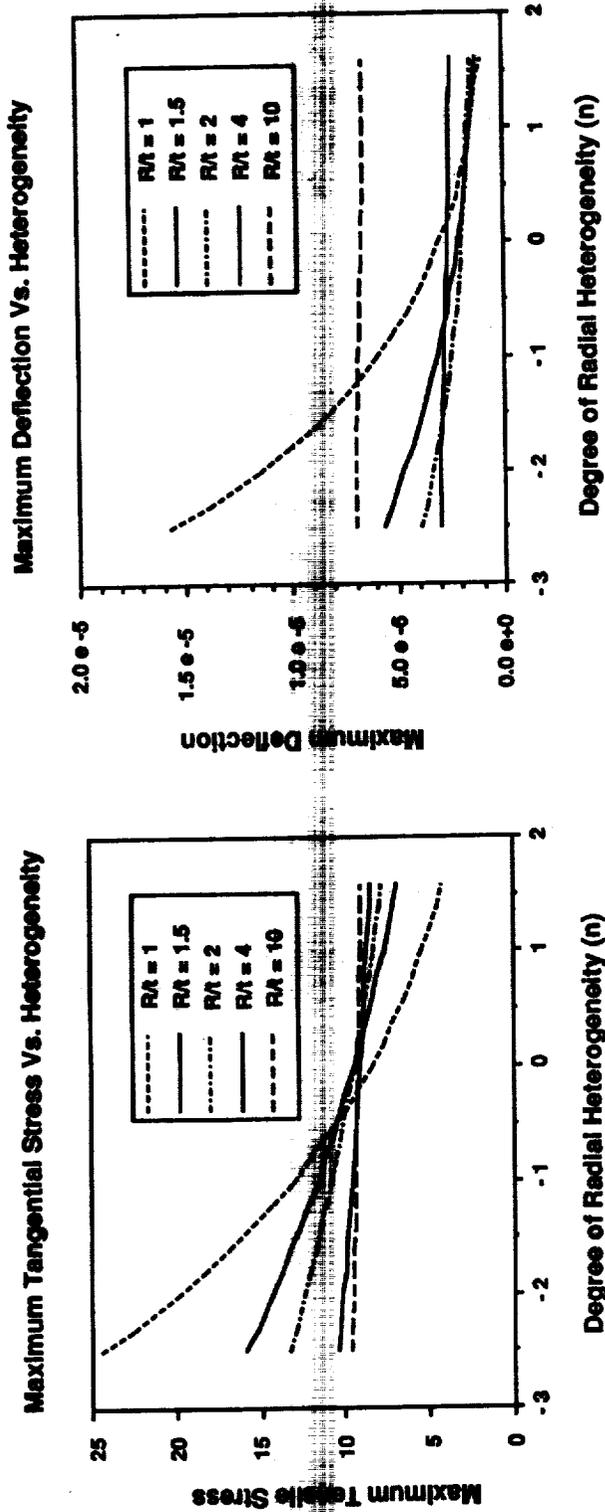


Figure 6: Maximum Tangential Stress vs. Heterogeneity and Maximum Displacement vs. Heterogeneity

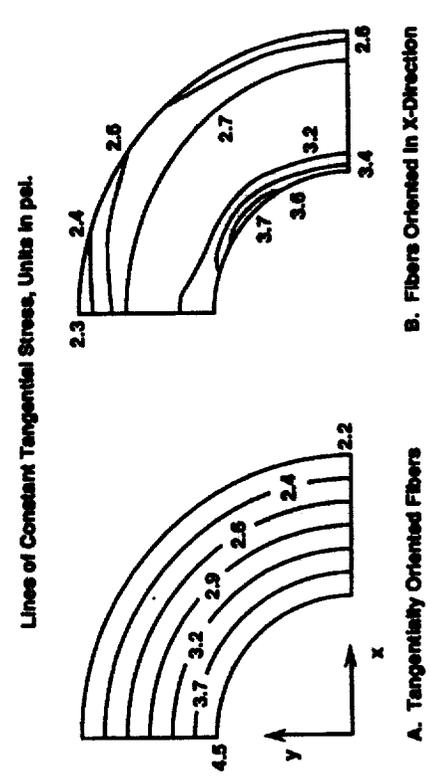


Figure 7. Tangential Stress Contours in a Circular Ring Loaded by an Internal Pressure,  $P_1 = 1$  psi.